Pergamon

Int. J. Solids Structures Vol. 35, Nos 31–32, pp. 4167–4189, 1998

0020–7683/98 $19.00 + .00

PII: S0020–7683(97)00308–9

REQUIREMENTS FOR ACCURATE QUANTIFICATION OF SELF-AFFINE ROUGHNESS USING THE VARIOGRAM METHOD

P. H. S. W. KULATILAKE,* J. UM and G. PAN
Department of Mining & Geological Engineering University of Arizona, Tucson, AZ 85721, U.S.A.

(*Received* 25 *September* 1996; *accepted* 16 *June* 1997)

Abstract—Both stationary and non-stationary fractional Brownian profiles (self-affine profiles) with known values of fractal dimension, D, input standard deviation, σ, and data density, d, were generated. For different values of the input parameter of the variogram method (lag distance, h), D and another associated fractal parameter K_v were calculated for the aforementioned profiles. It was found that σ has no effect on calculated D. The estimated K_v was found to increase with D, σ and d according to the equation $K_v = 2.0 \times 10^{-5} d^{0.35} \sigma^{1.95} D^{14.5}$. The parameter K_v seems to have potential to capture the scale effect of roughness profiles. Suitable ranges for h were estimated to obtain computed D within $\pm 10\%$ of the D used for the generation and also to satisfy a power functional relation between the variogram and h. Results indicated the importance of removal of non-stationarity of profiles to obtain accurate estimates for the fractal parameters. It was found that at least two parameters are required to quantify stationary roughness; the parameters D and K_v are suggested for use with the variogram method. In addition, one or more parameters should be used to quantify the non-stationary part of roughness, if it exists; at the basic level, the mean inclination/declination angle of the surface in the direction considered can be used to represent the non-stationarity. A new concept of feature size range of a roughness profile is introduced in this paper. The feature size range depends on d, D and σ. The suitable h range to use with the variogram method to produce accurate fractal parameter values for a roughness profile was found to depend on both d and D. It is shown that the feature size range of a roughness profile plays an important role in obtaining accurate roughness parameter values with both the divider and the variogram methods. The minimum suitable h was found to increase with decreasing d and increasing D. Procedures are given in this paper to estimate a suitable h range for a given natural rock joint profile to use with the variogram method to estimate D and K_v accurately for the profile.

1. INTRODUCTION

Strength, deformability and flow properties of joints depend very much on the surface roughness of joints. These effects arise from the fact that the surfaces composing a joint are rough and mismatched at some scale. The shape, size, number, and strength of contacts between the surfaces control the mechanical properties. The separation between the surfaces or the "aperture" determines the hydraulic properties. Therefore, accurate quantification of roughness is important in modeling strength, deformability and fluid flow behaviours of joints. Rock mass strength, deformability and fluid flow behaviours in turn depend very much on the properties of joints.

The Joint Roughness Coefficient, JRC (Barton, 1973), various conventional statistical parameters (Wu and Ali, 1978; Krahn and Morgenstern, 1979; Tse and Cruden, 1979; Dight and Chiu, 1981; Maerz *et al.*, 1990; Reeves, 1990) and fractal parameters (Brown and Scholz, 1985; Miller *et al.*, 1990; Power and Tullis, 1991; Huang *et al.*, 1992; Odling, 1994; Den Outer *et al.*, 1995; Kulatilake *et al.*, 1995) have been suggested for quantification of roughness of rock joints using linear profiles. The limitations of JRC and the conventional statistical parameters in quantifying joint roughness were mentioned in a previous paper (Kulatilake *et al.*, 1995). The same paper discussed the controversial findings which appear in the literature about using only the fractal dimension (Mandelbrot, 1983) to quantify rock joint roughness. A number of methods have been suggested in the literature to estimate D of roughness profiles of a rock joint surface. They are the divider (Mandelbrot, 1967),

* Corresponding author.

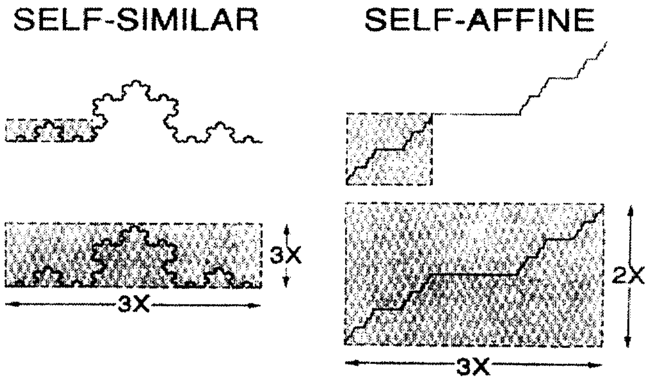


Fig. 1. Illustration of a self-similar and self-affine fractals.

box counting (Feder, 1988), variogram (Orey, 1970), spectral (Berry and Lewis, 1980), roughness-length (Malinverno, 1990), and the line scaling (Matsushita and Ouchi, 1989) methods.

Fractals can be either self-similar or self-affine. A self-similar fractal is a geometric feature that retains its statistical properties through various magnifications of viewing. A self-affine fractal remains statistically similar only if it is scaled differently in different directions. Figure 1 illustrates the concepts of self-similarity and self-affinity. It is very difficult for natural rock joint profiles to satisfy the requirements of self-similarity. Therefore, in general, it is not appropriate to consider natural rock joint profiles to be self-similar. However, they have the potential to be self-affine. The divider and the box counting methods work well for self-similar profiles. Their applicability to self-affine profiles is highly questionable at present (Kulatilake *et al.*, 1995). Part of the controversial findings, which appear in the literature, has resulted from application of the divider and the box methods in quantifying rock joint roughness profiles. The variogram, spectral, roughness-length, and the line scaling methods seem suitable to apply to self-affine profiles. However, it seems that the fractal parameters calculated by each of these methods depend significantly on the input parameter values used in each method, as well as on the stationary or non-stationary nature of the profile. It is possible that some of the controversial findings, which appear in the literature, might have resulted for the latter reasons. For stationary profiles, the mean and the variance do not vary with the spatial location and the auto-correlation function depends only on the lag distance, and not on the spatial location. In order to find explanations for the controversial findings which appear in the literature, it is necessary to investigate the effect of input and profile parameter values on the accuracy of fractal parameters calculated using each of these methods. Three recent papers have investigated the accuracy of fractal parameters estimated through the spectral (Shirono and Kulatilake, 1997), roughness-length (Kulatilake and Um, 1997), and line scaling (Kulatilake *et al.*, 1997) methods. This paper shows how such an investigation was performed for the variogram method, and presents the results and conclusions obtained.

2. MINIMUM AND MAXIMUM FEATURE SIZES OF A LINEAR ROUGHNESS PROFILE

Linear roughness of natural rock joint profiles can be measured either using a mechanical profilometer or a laser profilometer. Each profilometer has a smallest horizontal step at which the height of the roughness profile can be measured. Therefore, even though the roughness profiles of a natural rock joint surface are continuous, roughness profile data obtained through a profilometer are available only at a certain interval of horizontal spacing. When these roughness data are plotted, they may produce a profile as shown in Fig. 2. In this profile, the adjacent data points are connected through linear segments. Event though the horizontal length of each segment is the same, the inclined length (length of the segment) changes from one segment to another, depending on the inclination angle of the segment. Then the minimum feature size of a profile may be defined as the minimum

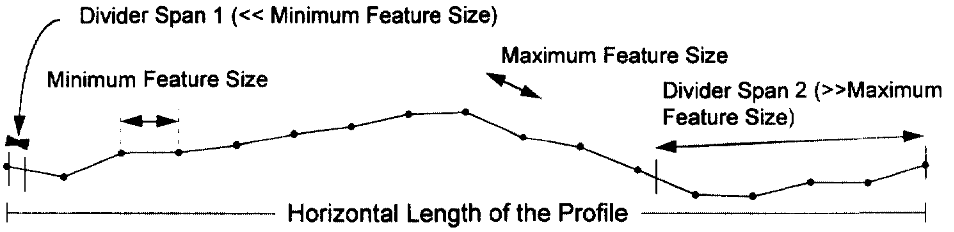


Fig. 2. Concept of minimum and maximum feature sizes.

segment length out of all the segment lengths between two adjacent data points on the profile (Fig. 2). This minimum distance cannot be less than the horizontal distance at which roughness height data are available. The maximum feature size may be defined as the maximum segment length out of all the segment lengths between two adjacent data points on the profile (Fig. 2). The difference between the maximum feature size and the minimum feature size of a profile decreases as the profile gets smoother. Also, it is important to realize that both the estimated minimum and the maximum feature sizes of a profile depend upon the resolution of the instrument used in measuring roughness.

The concepts mentioned above on the minimum and the maximum feature sizes are equally applicable for generated roughness profiles, because the generated values are available only at a certain interval of horizontal spacing. In the next section, it is shown that both the minimum and the maximum feature sizes of a roughness profile have important roles to play related to the accuracy of estimated D.

3. ILLUSTRATION OF THE INFLUENCE OF MINIMUM AND MAXIMUM FEATURE SIZES OF A ROUGHNESS PROFILE ON THE ACCURACY OF ESTIMATED FRACTAL DIMENSION USING THE DIVIDER METHOD

As pointed out earlier, irrespective of whether it is a natural rock joint or a generated roughness profile, roughness data of a profile are usually available at a certain interval of horizontal spacing. That allows one to estimate minimum and maximum feature sizes of a given profile for that horizontal spacing. Possible influence of these feature sizes on the estimated fractal dimension is shown in this section using the divider method.

The divider method is best visualized by considering a pair of dividers set to a particular span and then walked along the roughness profile. The number of divider steps required to cover the entire profile is counted, and then multiplied by the divider span, r, to give an estimate of the profile length, L. The divider span is set to another value and the process is repeated several times to produce a discrete relation between r and L. For self-similar fractals, the two are related linearly in log-log space according to the expression (Feder, 1988)

$$\log L = \log a + (1 - D) \log r \tag{1}$$

where log a is the intercept of the log L–log r plot. The slope of the log-log plot equals $1 - D$.

If the divider span is considerably shorter than the minimum feature size (for example, see divider span 1 shown in Fig. 2), that span will virtually trace the profile without bridging any peaks or valleys of the profile, returning the maximum possible value for the length. Therefore, for divider spans which are considerably shorter than the minimum feature size, the returned length L will be more or less the same. Due to this, the log L–log r curve gradually flattens as shown in Fig. 3 as r decreases beyond the minimum feature size. When the divider span is considerably larger than the maximum feature size (for example, see divider span 2 shown in Fig. 2), the returned length will be close to the horizontal length of the profile. Therefore, for the divider spans which are considerably larger than the maximum feature size, the returned lengths will be very close to the horizontal length of the profile. Due to this, the log L–log r curve gradually flattens as shown in Fig. 3 as r

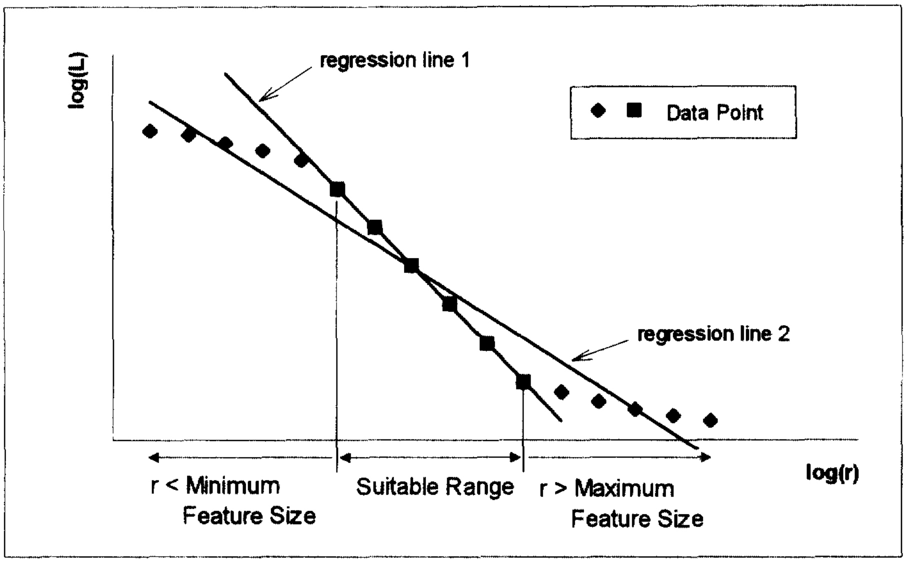


Fig. 3. Suitable range of r for the estimation of fractal dimension with the divider method.

increases beyond the maximum feature size. The aforementioned facts show the difficulty of obtaining a unique slope for the log L–log r relation for the whole range of r as shown in Fig. 3. The correct slope of log L–log r and thus the correct D can be obtained by fitting a regression line to the log L–log r data in the non-flattening portion of the curve (i.e. regression line 1 in Fig. 3). The above discussion indicates very clearly that there is a need to select a suitable range for r, which is the input parameter in the divider method, taking into account the minimum and the maximum feature sizes in order to obtain accurate D values. Values of r considerably smaller than the minimum feature size or considerably larger than the maximum feature size will produce erroneous D values which are almost 1.

Similarly, for other methods which are used to estimate D, input parameters might have to be in certain ranges in order to obtain accurate D values. This paper examines that aspect in a systematic fashion for the variogram method.

4. GENERATION OF FRACTIONAL BROWNIAN FUNCTION PROFILES

Classical examples for self-affine profiles are the fractional Brownian functions (Saupe, 1988; Voss, 1988). Several methods are available to generate the fractional Brownian profiles. The two most popular methods are the random midpoint displacement method (Fournier *et al.*, 1982) and the spectral synthesis method (Fox, 1987; Saupe, 1988; Voss, 1988).

In this study, the random midpoint displacement method was used to generate fractional Brownian profiles. First, the coordinate axes x and y are selected along the horizontal and vertical directions, respectively (Fig. 4). Then, two points (points 1 and 2 in Fig. 4) are selected at a certain distance apart to represent the starting and ending points of the profile. By selecting these two points at the same vertical level, stationary profiles can be generated. By choosing the above two points at different vertical levels, a non-stationary profile with a trend can be generated. The x coordinate at point 3, $x(3)$, is selected as $(1/2)[x(1)+x(2)]$. The y coordinate at point 3, $y(3)$, is generated as

$$y(3) = \tfrac{1}{2}[y(1)+y(2)]+D_1 \qquad (2a)$$

where D_1 is normally distributed with mean zero and a variance Δ_1^2 given as

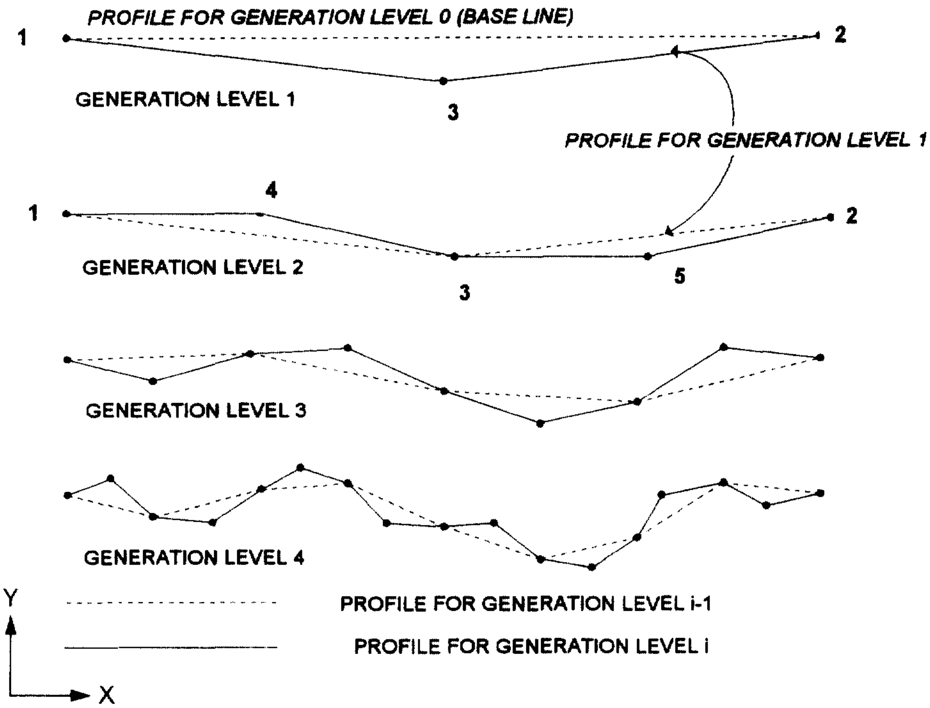


Fig. 4. Generation of a stationary fractional Brownian profile using the midpoint displacement method.

$$\Delta_1^2 = \frac{\sigma^2(1-2^{2H-2})}{(2)^{2H}} \tag{2b}$$

where σ and H $(0 < H < 1)$ are, respectively, the initial standard deviation and the Hurst exponent used in the generation. Note that D is related to H through $D = 2-H$. Figure 4 shows a typical generation for point 3. Then, at the second generation level, points 4 and 5 are generated according to the following equations:

$$x(4) = \tfrac{1}{2}[x(1)+x(3)] \tag{3a}$$

$$y(4) = \tfrac{1}{2}[y(1)+y(3)]+D_2 \tag{3b}$$

$$x(5) = \tfrac{1}{2}[x(3)+x(2)] \tag{3c}$$

$$y(5) = \tfrac{1}{2}[y(3)+y(2)]+D_2 \tag{3d}$$

where D_2 is normally distributed with mean zero and a variance Δ_2^2 given as

$$\Delta_2^2 = \frac{\sigma^2(1-2^{2H-2})}{(2^2)^{2H}} \tag{3e}$$

The above procedure is repeated n times, to obtain a stationary fractional Brownian profile at the nth generation level, having the selected D or H and σ values. Note that the values for two parameters are required to generate a fractional Brownian profile. At the nth generation level, D_n is normally distributed with mean zero and a variance Δ_n^2 given by

$$\Delta_n^2 = \frac{\sigma^2(1-2^{2H-2})}{(2^n)^{2H}} \tag{4}$$

At the nth generation level, the profile has 2^n+1 data points. The stationary Brownian profiles generated for $D = 1.2$ through 1.7 with $\sigma = 5$ at the eleventh generation level are shown in Fig. 5. At the eleventh generation level, the total profile has 2049 data points. The total horizontal length of the profile is taken as 200 units. Therefore, each of these profiles has a data density, d, of 10.25 per unit length. All these profiles were generated with a starting seed value of 1.

5. MAIN FEATURES OF THE VARIOGRAM METHOD

Let $Z(x)$ be a Gaussian process with stationary increments, mean = 0, and the variogram function given by $2\gamma(x, h) = E[(Z(x+h)-Z(x))^2]$ where h is the lag distance along the x-axis. If $2\gamma(x, h)$ behaves like h^{2H} as $h \to 0$ (where H is the Hurst exponent), then the fractal dimension, D, of $Z(x)$ is equal to $2-H$ (Orey, 1970). To estimate D, first H should be estimated. Before H is estimated, it is necessary to check whether the following power law equation holds true:

$$2\gamma(x, h)_{h\to 0} = K_v h^{2H} \tag{5}$$

where K_v is a proportionality constant. This can be evaluated by checking the linearity of the plot between $\log(\text{variogram})_{h\to 0}$ and $\log(h)$. Note that the slope of log(variogram) and $\log(h)$ plot is equal to $2H$.

Let x be the horizontal distance along a roughness profile and $Z(x)$ be the height of the roughness profile from the datum. To calculate the variogram for digitized roughness profile data, the variogram is expressed in the discretized form as

$$2\gamma(x, h) = \frac{1}{M}\sum_{i=1}^{M}[Z(x_i)-Z(x_i+h)]^2 \tag{6}$$

where M is the total number of pairs of roughness heights of the profile that are spaced at a lag distance h. The variogram given by eqn (6) is termed the experimental variogram.

Because $H = 2-D$, eqn (5) can be written as

$$2\gamma(x,h)_{h\to 0} = K_v h^{2(2-D)} \tag{7}$$

From eqn (7), it is clear that the variogram is well related to the roughness. For the same lag distance h, the higher the $2\gamma(x, h)$, the rougher the profile. Equation (7) shows that the roughness is not only related to D, but also to K_v and h. Note that $\log K_v$ is equal to the intercept of log(variogram) vs $\log h$. In eqn (7), since $2\gamma(x, h)$ and $h^{2(2-D)}$ are positive, then K_v should be positive. If K_v is a constant and $K_v > 0$, then when $h < 1$, the higher the D, the rougher the profile. However, when $h > 1$, the higher the D, the smoother the profile. In addition, K_v need not be a constant. Therefore, it is clear that D along cannot quantify roughness. The hypothesis will also be clear from the next section. When $h = 1$ unit, $2\gamma(x, h) = K_v$. The unit of h can be changed from mm to km depending on the scale of the roughness profile. Therefore, the value of K_v can change depending on the unit chosen to represent h. This means K_v has the potential to capture the scale effect of roughness.

6. EFFECT OF INPUT AND PROFILE PARAMETERS ON ACCURACY OF D

6.1. *Influence of* D *on suitable* h *range*

The generated profiles shown in Fig. 5 have the following properties: (a) horizontal profile length = 200 units, (b) generation level = 11, (c) $d = 10.25$ per unit length, (d)

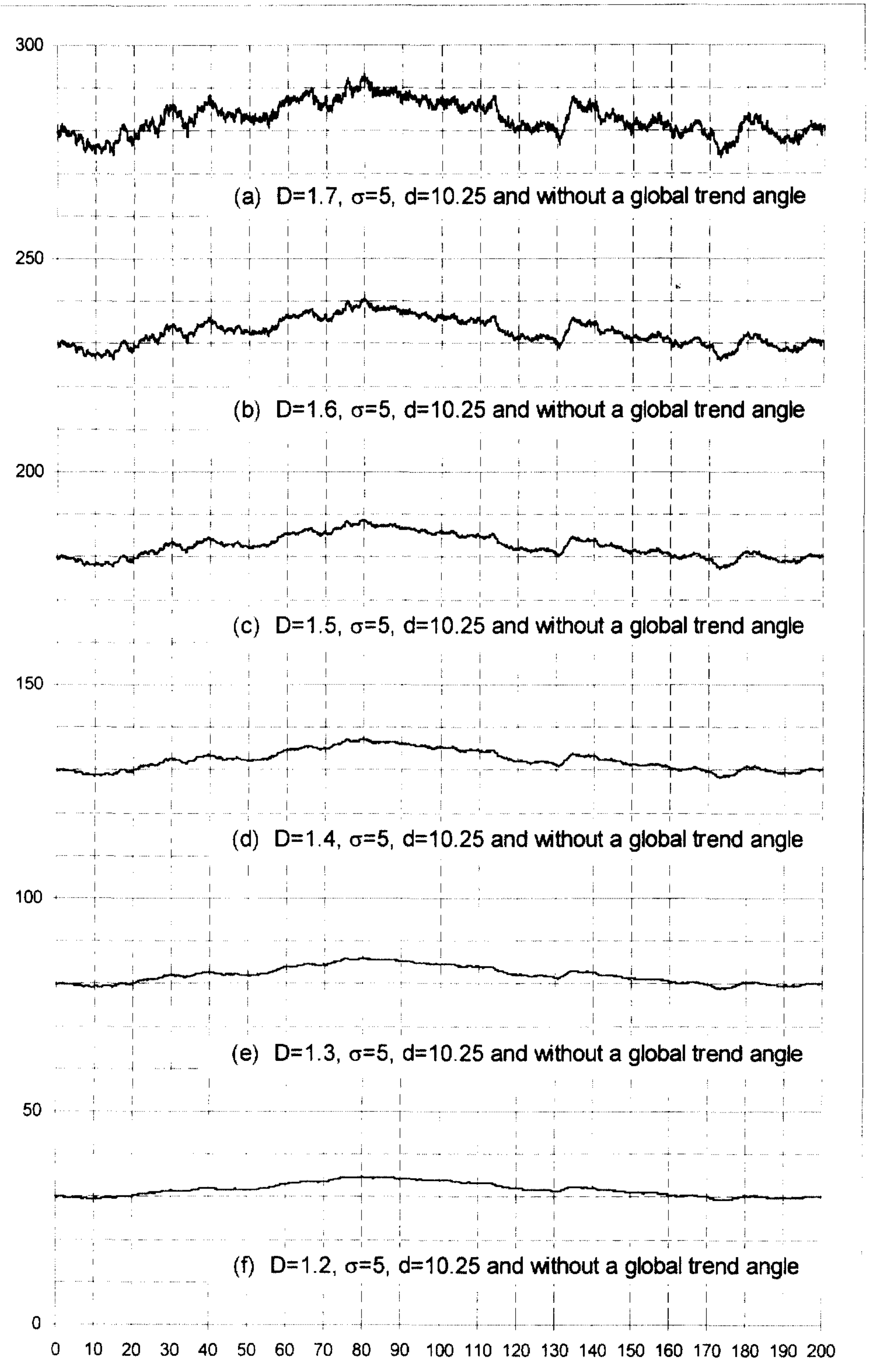


Fig. 5. Stationary fractional Brownian profiles at the 11th generation level with $\sigma = 5$, $d = 10.25$ and different D: (a) $D = 1.7$, (b) $D = 1.6$, (c) $D = 1.5$, (d) $D = 1.4$, (e) $D = 1.3$ and (f) $D = 1.2$.

minimum feature size = 0.0977 units, and (e) $\sigma = 5$. The maximum feature size of the profile increases with the D value used for the generation (Table 1).

In applying the variogram method to generated fractional Brownian profiles, the variogram was calculated for different h values starting from an initial h value and increasing

Table 1. Influence of D and σ on maximum feature size of stationary fractional Brownian profiles having $d = 10.25$ per unit length

D	Maximum feature size			
	$\sigma = 2.5$	$\sigma = 5$	$\sigma = 7.5$	$\sigma = 10$
1.7	1.439	2.873	4.308	5.743
1.6	0.725	1.441	2.158	2.876
1.5	0.369	0.719	1.073	1.428
1.4	0.199	0.361	0.530	0.701
1.3	0.127	0.191	0.264	0.342
1.2	0.105	0.124	0.151	0.182

the h sequentially by multiplying through an increment factor of 1.2. Figure 6 shows log(variogram) vs log(h) plots obtained for the profiles in Fig. 5. These plots show that for very small h values, log(variogram) vs log(h) is slightly non-linear. This range of non-linearity increases with D. The plots also show that for h values greater than 10 (i.e. log(h) = 1.0) the relation between log(variogram) and log(h) deviates from linearity. The amount of deviation seems to increase with D. The linearity aspect between the above two variables will be further investigated quantitatively. To estimate each pair of K_v and D through the plot between log(variogram) and log(h), seven consequent lag distances were used. The estimated K_v and D were assigned to the middle h value of each seven lag distances. Because, for each profile, the D used for the generation is known, the error percentage of estimated D can be calculated. Figure 7 shows the plots between error percentage of estimated D and h for the profiles shown in Fig. 5. In the same plots, the

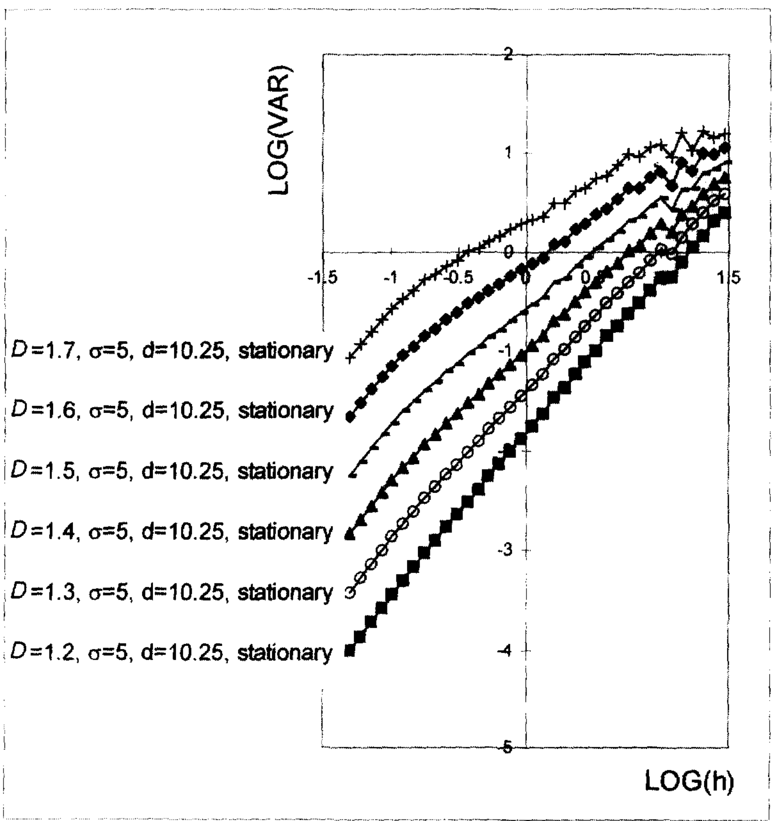
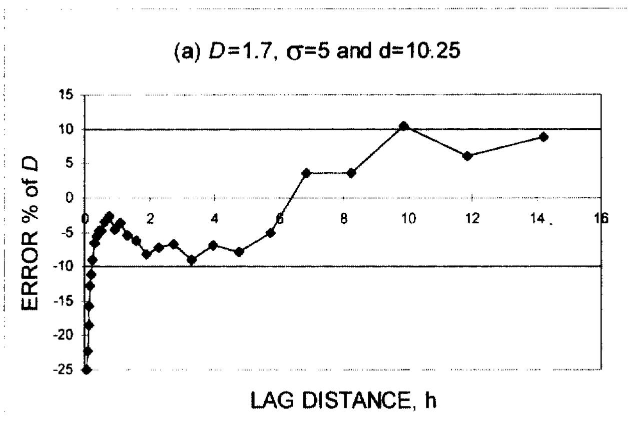


Fig. 6. Log(variogram) vs log(h) plots obtained for the stationary fractional Brownian profiles having $\sigma = 5$, $d = 10.25$ and different D.

expected D and the $\pm 10\%$ error bounds are also shown to determine the accuracy of the estimated D. Figure 8 shows the range of h corresponding to each D to obtain estimated D within $\pm 10\%$ of the expected D for the profiles having $d = 10.25$ and $\sigma = 5$. It is clear from Fig. 8 that the minimum suitable h to produce D within $\pm 10\%$ error increases with D, even though the variation is small. Note that this behaviour is compatible with the non-linearity behaviour observed for log(variogram) vs log(h) for small h values. Figure 8 also shows that there exist a tendency for maximum suitable h to produce D within $\pm 10\%$ error to increase with D.

The linearity of each log(variogram) vs. log(h) plot was examined by computing the correlation coefficient, R $(0 < R < 1)$, and the normalized standard error estimates. R values closer to 1 and low normalized standard errors indicate strong linear relations. R values greater than 0.93 and normalized standard errors less than 4% were obtained for h ranges between the minimum suitable h (to produce D within $\pm 10\%$ error) and a maximum h of 5.72 for all the D values investigated. These R and normalized standard error values indicate strong linear relations. Note that 5.72 is the lowest value obtained for the maximum suitable h to produce D within $\pm 10\%$ error among all the profiles investigated with $d = 10.25$ and different D values. When $5.72 < h \leqslant 8.24$, R values between 0.56 and 0.91 and normalized standard error values between 5.6% and 8.0% were obtained for $1.4 \leqslant D \leqslant 1.7$. Some of these R and normalized standard error values do not show strong linear correlations. That implies inapplicability of eqn (5). For $D = 1.3$, normalized error values up to 4.7% were obtained when $5.72 < h \leqslant 6.87$. According to the above findings, it can be



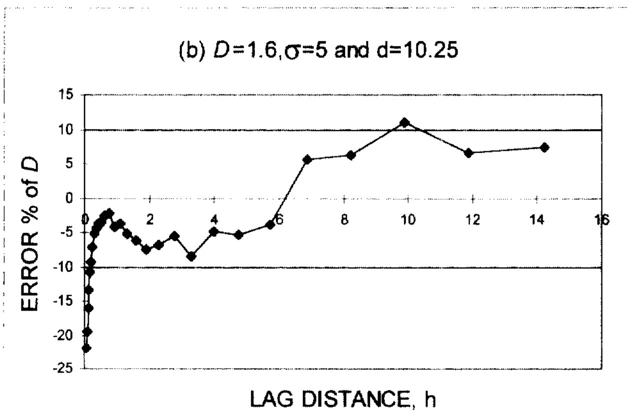


Fig. 7. Plots between percentage error of D and lag distance, h, for stationary fractional Brownian profiles having $\sigma = 5$, $d = 10.25$ and different D: (a) $D = 1.7$, (b) $D = 1.6$, (c) $D = 1.5$, (d) $D = 1.4$, (e) $D = 1.3$ and (f) $D = 1.2$.

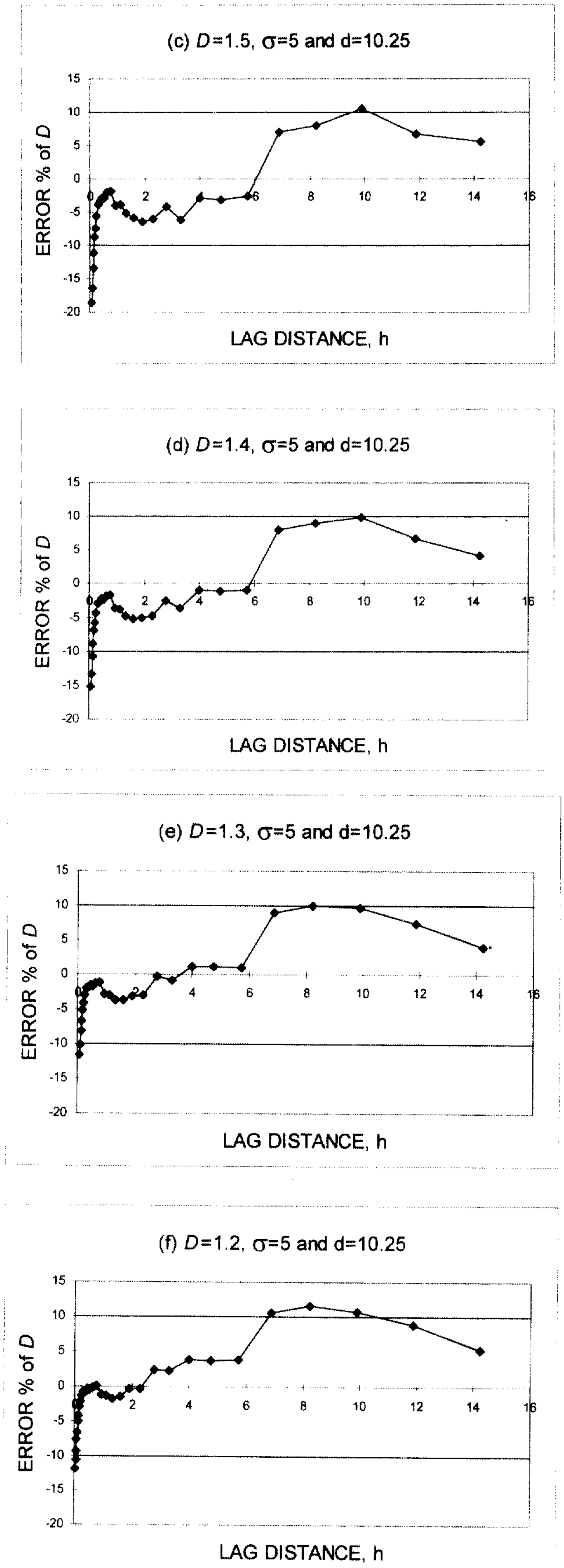


Fig. 7.—*continued*

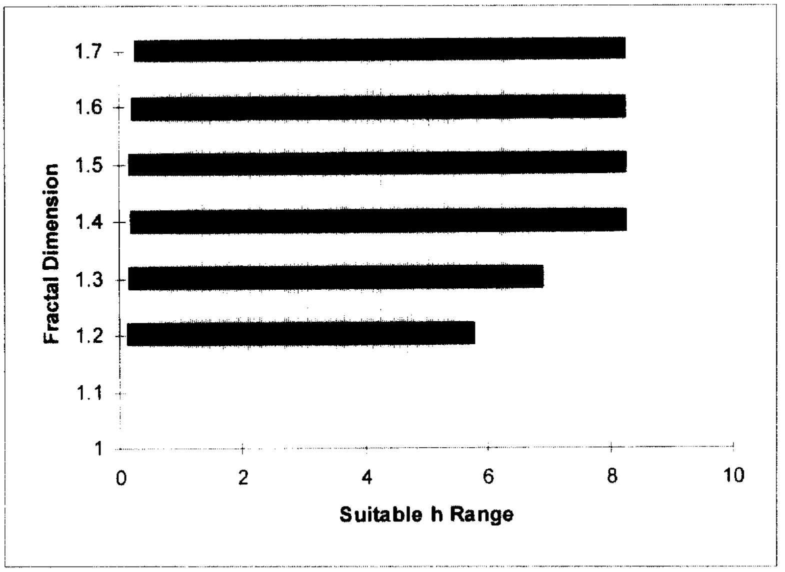


Fig. 8. Relation between h range to produce D within $\pm 10\%$ error and fractal dimension for stationary fractional Brownian profiles having $d = 10.25$ and $\sigma = 5$.

concluded that the suitable h range to obtain accurate D for stationary fractional Brownian profiles having $d = 10.25$ and $1.2 \leqslant D \leqslant 1.7$ is between the minimum suitable h to produce D within $\pm 10\%$ and 5.72. In Section 6.4, it will be shown that these suitable h ranges change according to the d value.

The summary statistics for the calculated K_v in the suitable range of h for profiles with different D and $\sigma = 5$ are given in Table 2. Table 2 shows that K_v increases with D when σ is kept constant.

6.2. *Effect of* σ

All the estimations mentioned in the previous section were performed on stationary fractional Brownian profiles generated with $\sigma = 5$. In order to study the effect of σ on the estimated parameters, a few more stationary fractional Brownian profiles were generated

Table 2. Effect of D and σ of fractional Brownian profiles with $d = 10.25$ on estimated K_v

Input parameter values for fractional Brownian profiles		Summary statistics for K_v				
D	σ(STD)	Mean	Standard deviation	Coefficient of variation	Minimum	Maximum
1.7	2.5	0.507	0.036	0.070	0.429	0.601
1.5	2.5	0.070	0.005	0.075	0.065	0.085
1.3	2.5	0.011	0.001	0.079	0.010	0.013
1.7	5	2.027	0.142	0.070	1.716	2.402
1.6	5	0.742	0.055	0.074	0.658	0.910
1.5	5	0.280	0.021	0.075	0.261	0.339
1.4	5	0.108	0.009	0.080	0.101	0.135
1.3	5	0.042	0.003	0.079	0.040	0.052
1.2	5	0.016	0.002	0.122	0.023	0.015
1.7	7.5	4.561	0.320	0.070	3.860	5.405
1.5	7.5	0.629	0.047	0.075	0.586	0.763
1.3	7.5	0.095	0.008	0.079	0.089	0.117
1.7	10	8.330	0.604	0.073	7.616	9.804
1.5	10	1.119	0.084	0.075	1.043	1.357
1.3	10	0.169	0.013	0.079	0.158	0.209

with the following properties: (a) $D = 1.5$, (b) horizontal profile length = 200 units, (c) generation level = 11, (d) 10.25 per unit length, and (e) minimum feature size = 0.0977 units and $\sigma = 2.5$, 7.5, and 10 (Figs 9a–9d). The necessary calculations were performed on the profiles shown in Figs 9a–9d to estimate (a) maximum feature size, (b) D, (c) suitable h ranges to obtain accurate D, and (d) K_v. The maximum feature size was found to increase with σ (Table 1). The error percentage of estimated D vs log(h) plots obtained for $\sigma = 2.5$, 5.0, 7.5 and 10 turned out to be exactly the same. Similar findings were obtained for profiles having other D values and same d. These results indicate that σ has no influence on estimated D and suitable h range. Figure 10 shows the log(variogram) vs log(h) plots obtained for the profiles shown in Figs 9a–9d. These plots show that K_v increases with σ. Similar findings were obtained for profiles with other D values having the same d. Table 2 shows that K_v increases significantly with σ when D is kept constant. Also, the table indicates that the effect of σ on K_v increases with increasing D. Note also the low coefficient of variation values obtained for K_v (Table 2) indicating high reliability for estimated K_v.

6.3. *Non-stationary effect*

All the profiles considered so far can be considered as stationary profiles. A stationary profile has a constant mean and a variance, and an auto-correlation function which only depends upon the lag distance irrespective of the spatial location. In order to study the non-stationary effect on the estimated parameters, a new set of fractional Brownian profiles were generated with the following properties: (a) $D = 1.5$, (b) horizontal profile length = 200 units, (c) generation level = 11, (d) $d = 10.25$ per unit length, (e) minimum feature size = 0.0977 units, (f) $\sigma = 5$, and (g) trend angles of the profiles = 5 and 10 degrees (Figs 9e and f). For these non-stationary profiles, calculations were performed similar to the calculations performed on the profiles shown in Fig. 5 to estimate the same parameters.

The maximum feature sizes of the generated non-stationary profiles with $D = 1.5$, $\sigma = 5$ and the trend angles of 5 and 10 degrees turned out to be around 2.864 and 2.856, respectively. Table 1 shows that the maximum feature size of the stationary profile with $D = 1.5$, $\sigma = 5$ and trend angle = 0° is 2.8728. This indicates that the non-stationary effect resulting from a global trend has negligible influence on the feature size range of a roughness profile. Figure 11 shows plots between error % of estimated D and lag distance h for profiles having $D = 1.5$, $\sigma = 5$ and different trend angles. The figure clearly indicates that the calculated D can become inaccurate in the presence of a non-stationary portion of a roughness profile. This instability of D in the non-stationary presence is also depicted in Fig. 10. Similar findings were obtained for profiles with other D and σ values. Therefore, for non-stationary profile, a suitable range for h does not exist and also the calculated K_v is inaccurate. To obtain accurate D and K_v, the stationary part of the non-stationary profile should be used.

6.4. *Influence of data density on suitable* h *range*

Data density of fractional Brownian profiles can vary either due to a change in the generation level, GL, or due to a change in the profile length, L. Effect of GL as well as L on suitable h range is investigated in this section. In Section 6.1, it was concluded that the minimum suitable h depends on the D value. On the other hand, it was concluded in Section 6.2 that σ has no influence on the suitable h range. Therefore, to study the influence of data density, profiles with different D but having the same σ are considered.

6.4.1. *Influence of generation level* (GL) *on suitable* h *range*. Figure 12 shows the stationary fractional Brownian profiles having $D = 1.5$, $\sigma = 5$ and $L = 200$ for different generation levels. The data density corresponding to each generation level is also given in Fig. 12. The figure shows clearly how the details of a profile increase with increasing generation level. The plots between the error percentage of estimated D and the lag distance for the profiles shown in Fig. 12 are shown in Fig. 13. From Fig. 13 it is clear that the minimum suitable h increases with decreasing d. Figure 14 shows the developed function for this relation for $D = 1.5$ through regression analysis. Figure 14 also provides the

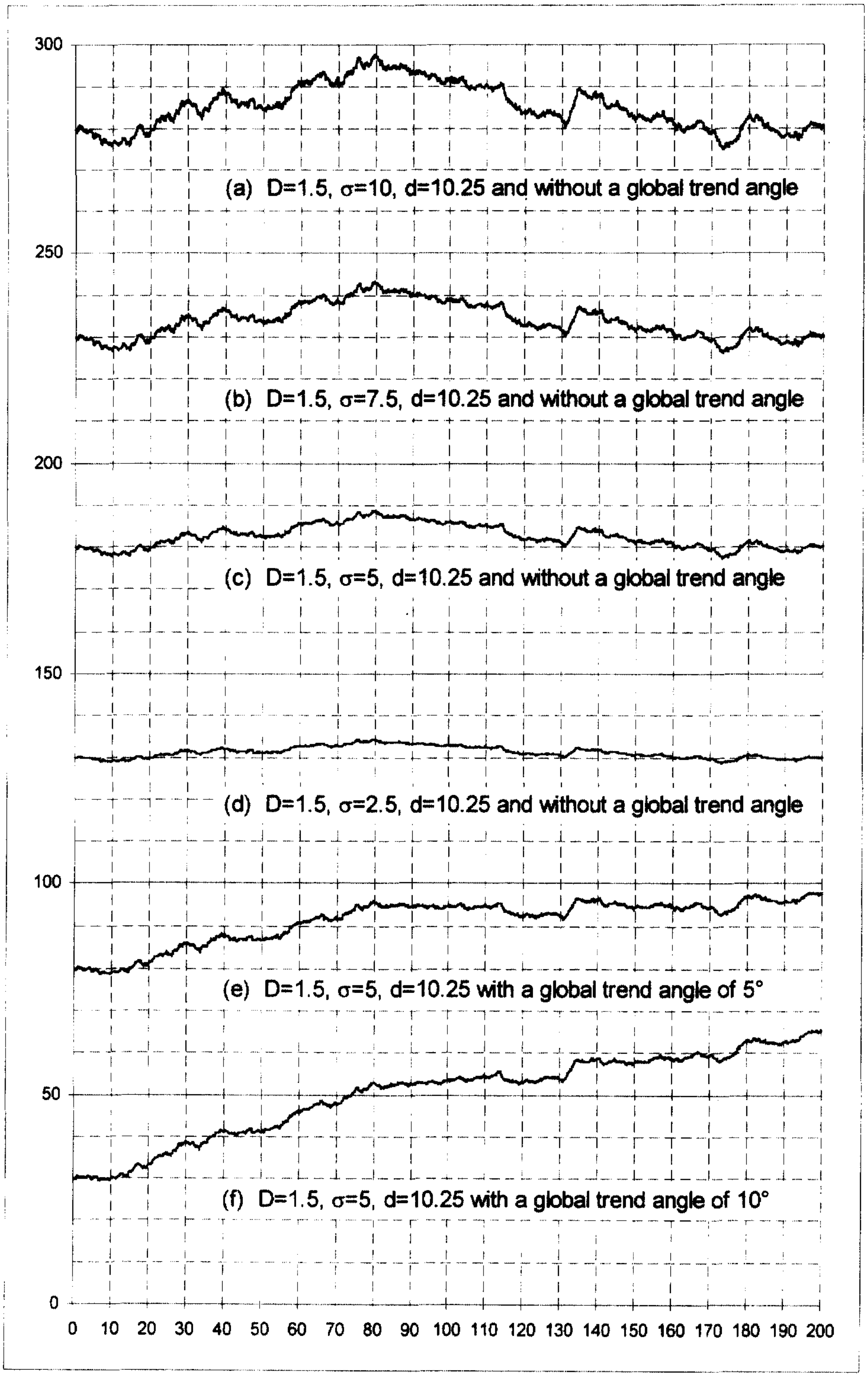


Fig. 9. Fractional Brownian profiles at the 11th generation level with $D = 1.5$, $d = 10.25$ and different σ and trend angles: (a) $\sigma = 10$ and trend angle $= 0°$, (b) $\sigma = 7.5$ and trend angle $= 0°$, (c) $\sigma = 5$ and trend angle $= 0°$, (d) $\sigma = 2.5$ and trend angle $= 0°$, (e) $\sigma = 5$ and trend angle $= 5°$ and (f) $\sigma = 5$ and trend angle $= 10°$.

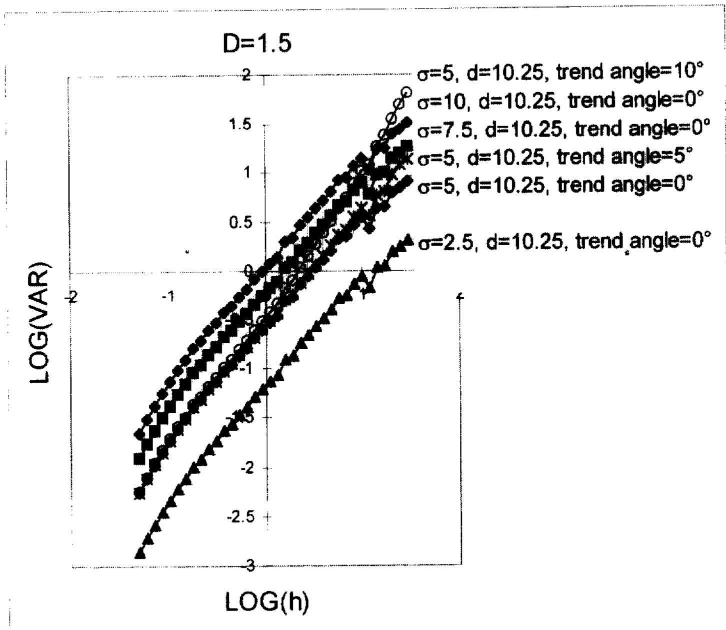


Fig. 10. Relation between log(variogram) and log(h) for fractional Brownian profiles having $D = 1.5$, $d = 10.25$ and different σ and trend angles.

developed functions resulting from profiles having $D = 1.2$ and $D = 1.7$ and different GL or d values.

The exponent values obtained for the three developed functions are very close to minus one. For all practical purposes, this developed relation for any D value can be considered as $hd =$ constant; the value of the constant increases with D. For natural rock joint profiles, D greater than 1.5 can be considered as a rare situation. If the equation applicable for $D = 1.5$ (i.e. $hd = 1.71$) is used to compute the minimum suitable h corresponding to the d value of a given natural rock joint profile, this h value will be greater than or equal to the minimum suitable h arising from profiles having $D \leqslant 1.5$. Therefore, the minimum suitable h computed through $hd = 1.71$ can be used conservatively for natural rock joint profiles having $D \leqslant 1.5$.

Figure 13 does not show a functional relation between the maximum suitable h and d. However, the figure shows that it is possible to have at least seven h values between the minimum suitable h and the maximum suitable h, if an increment of 1.2 is used for h. This information is useful in applying to natural rock joint profiles. Therefore, to apply to natural rock joint profiles, first the minimum suitable h can be calculated based on the developed equation applicable for $D = 1.5$; then, six more h values can be calculated using the obtained minimum h and the increment factor of 1.2. These seven h values then can be used in determining D and K_v for the given natural rock joint profile. It is important to note that the developed equation corresponding to $D = 1.5$ is still not in the final form. The final version of that equation is given at the end of Section 6.4.2 incorporating results of effect of d arising due to both GL and L.

6.4.2. *Influence of profile length* (L) *on suitable* h *range*. Figure 15 shows the stationary fractional Brownian profiles having $D = 1.5$, $\sigma = 5$, $GL = 11$, and different profile lengths. All these profiles have 2049 data points. The data density corresponding to each profile length is also given in Fig. 15. Each of these profiles was used to compute the minimum suitable h. The developed relation between the minimum suitable h and d using the results obtained for $D = 1.5$ is shown in Fig. 16. Note that again the exponent value in the relation is almost minus one. In addition, the constant value is quite close to the value obtained earlier for $D = 1.5$ using the profiles having constant L and different GL values. Therefore,

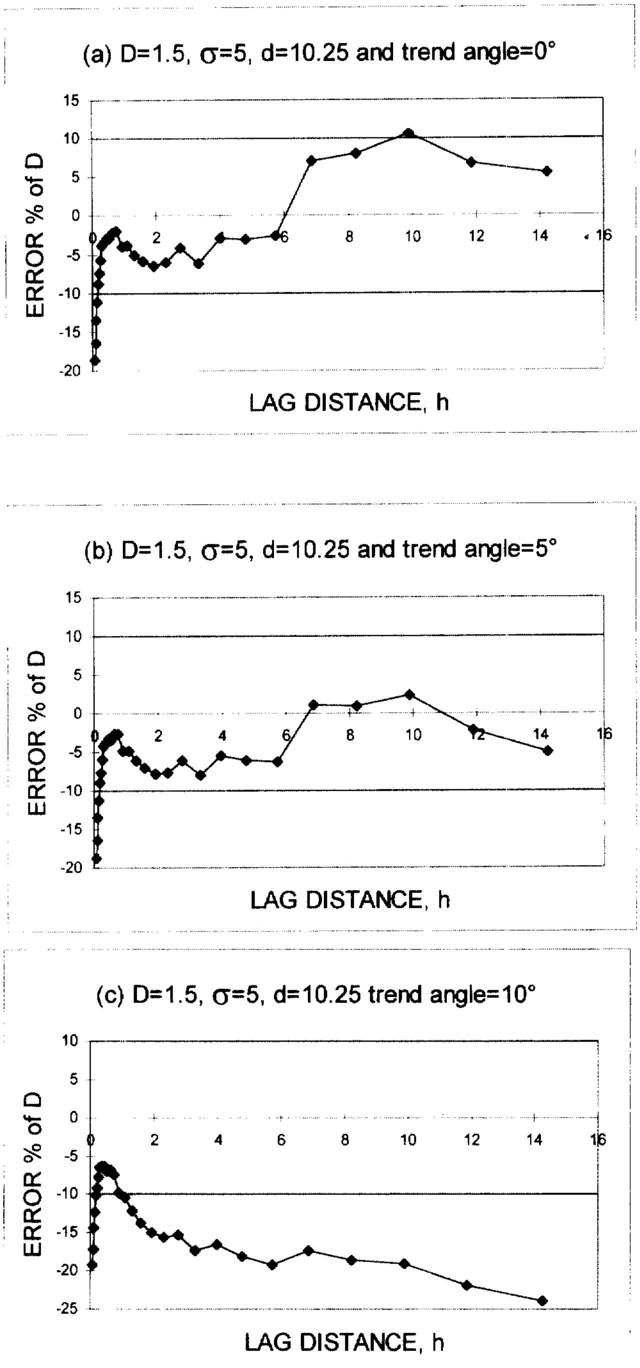


Fig. 11. Plots between percentage error of D and lag distance h for fractional Brownian profiles having $D = 1.5$, $\sigma = 5$, $d = 10.25$ and different trend angles: (a) trend angle $= 0^\circ$, (b) trend angle $= 5^\circ$ and (c) trend angle $= 10^\circ$.

it was decided to combine the data shown in Fig. 16 and the data corresponding to $D = 1.5$ in Fig. 14 and develop the final relation between the minimum suitable h and d for $D = 1.5$. This final relation is shown in Fig. 17. Note that the exponent value in the relation is almost minus one. Therefore, to apply to natural rock joint profiles, the following equation can be used to estimate the minimum or the starting suitable h by knowing the d value for the profile:

$$hd = 1.76 \tag{8}$$

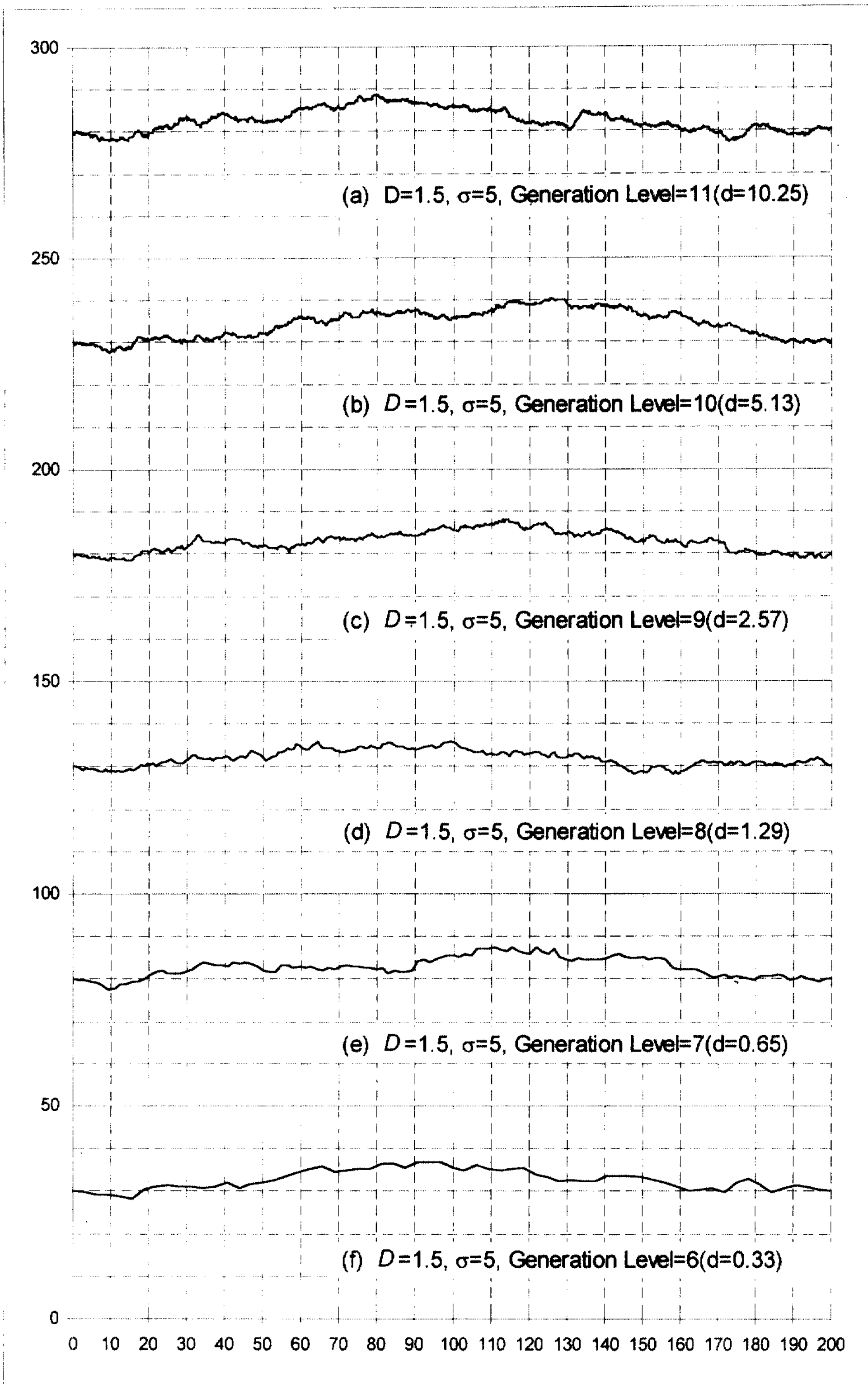


Fig. 12. Stationary fractional Brownian profiles with $D = 1.5$, $\sigma = 5$, $L = 200$ and different generation levels: (a) $GL = 11$, (b) $GL = 10$, (c) $GL = 9$, (d) $GL = 8$, (e) $GL = 7$ and (f) $GL = 6$.

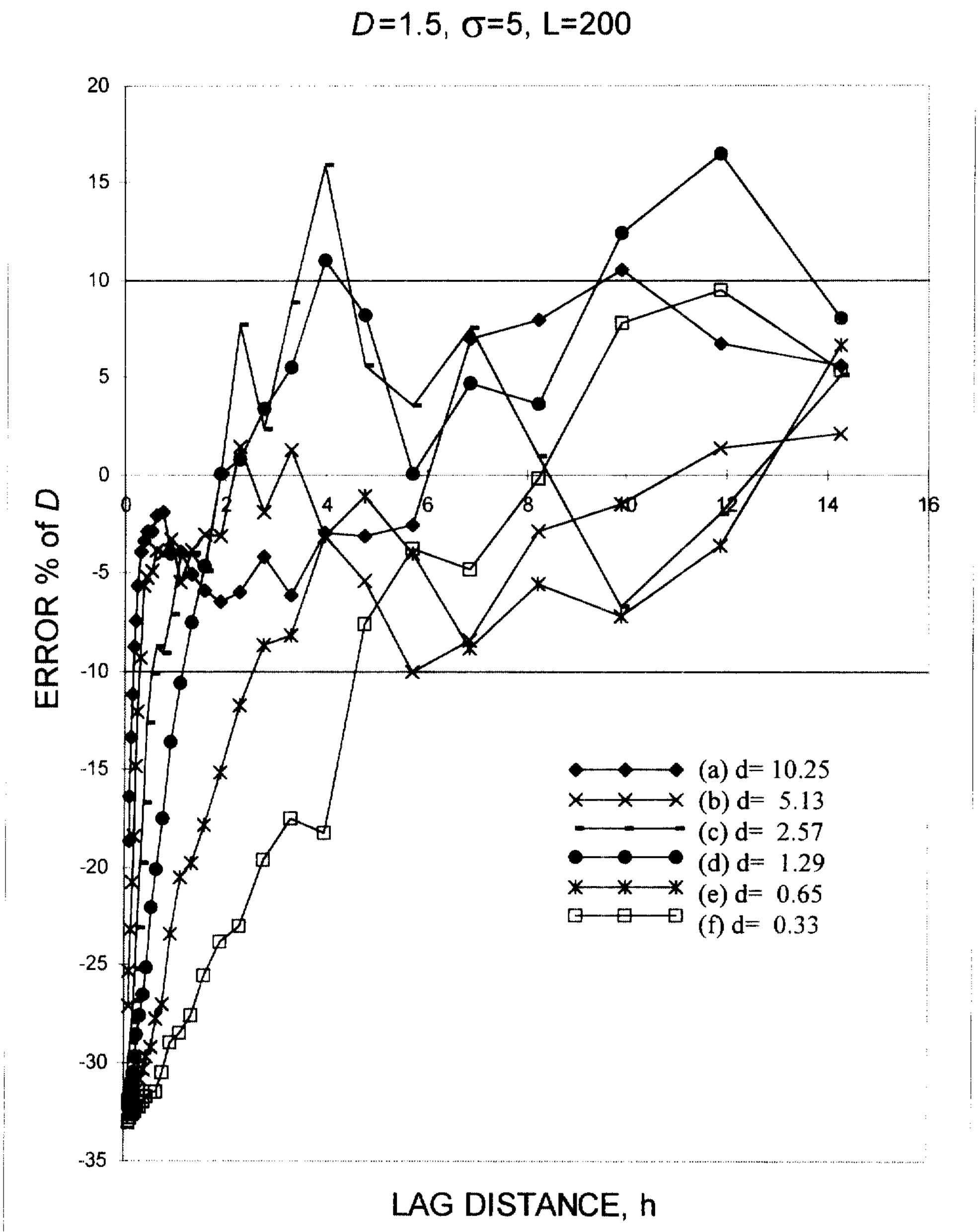


Fig. 13. Plots between percentage error of D and lag distance, h, for stationary fractional Brownian profiles having $D = 1.5$, $\sigma = 5$, $L = 200$ and different d (resulting from different GL): (a) $d = 10.25$, (b) $d = 5.13$, (c) $d = 2.57$, (d) $d = 1.29$, (e) $d = 0.65$ and (f) $d = 0.33$.

The next six h values then can be calculated using the calculated minimum suitable h and an increment factor of 1.2 for h. These seven h values then can be used in estimating D and K_v for the given natural rock joint profile.

6.5. *Effect of seed value on suitable* h *range*

In all the aforementioned investigations a seed value of 1 was used in generating the required fractional Brownian profiles. In order to investigate the effect of seed value on accuracy of D, several functional Brownian profiles having $D = 1.5$, $\sigma = 5$, $GL = 11$, $L = 200$, $d = 10.25$ were generated using different seed values (Fig. 18). Each of these

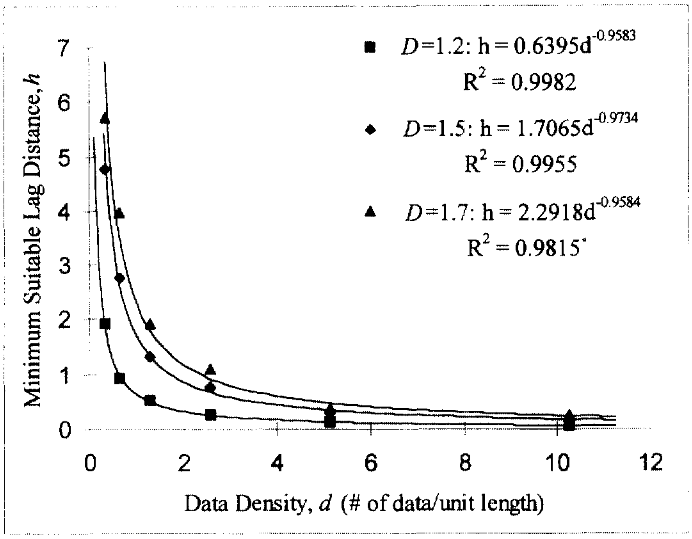


Fig. 14. Plots between minimum suitable lag distance and data density for stationary fractional Brownian profiles having $D = 1.2$, $D = 1.5$, $D = 1.7$, $L = 200$ and different GL.

profiles was used to estimate the suitable h range to satisfy an R value of greater than 0.85 and D to be within 10% error. The obtained results (Fig. 19) show that the seed value has negligible influence on suitable h range.

7. SUMMARY AND CONCLUSIONS

A new concept of feature size range of a roughness profile is introduced in the paper. Conceptually, it is shown that this feature size range plays a very important role in obtaining accurate D values using the divider method. A previous paper (Kulatilake *et al.*, 1995) has shown that accurate estimation of D is difficult for self-affine profiles with the divider method.

The fractional Brownian profiles considered in this research are quite similar to the roughness profiles which are observed for natural rock joints. The profiles with low D and σ especially seem to be very close to natural rock joint profiles. Therefore, the results obtained in this study are applicable in characterizing natural rock joint profiles. For a given natural rock joint profile, the minimum feature size depends on the data density of the profile. The maximum feature size of profiles increases with increasing D and σ for profiles having the same data density (Table 1). Therefore, for a natural rock joint profile, the maximum feature size depends on data density, D and σ values of the profile. The suitable h range obtained for the variogram method to produce accurate fractal parameter values was found to depend on both data density and D, but not on σ. Therefore, it is clear that the feature size range plays an important role in obtaining accurate estimates for fractal parameters through the variogram method.

The results obtained in this study clearly show that the variogram method cannot produce accurate estimates for fractal parameters if the method is applied directly to non-stationary profiles. However, if the method is applied to the stationary part of a non-stationary profile, it produces accurate estimates for fractal parameters. Therefore, when applying the variogram method to natural rock joint profiles, first it is necessary to remove the non-stationarity of the profile, if it exists.

The minimum suitable h value increases with decreasing d (Figs 14, 16 and 17). This variation is dramatic for $d < 1.0$ (Figs 14 and 17). Therefore, in applying to natural rock joint profiles, it is recommended to choose a unit of length so that d for a given profile becomes greater than 1. The minimum suitable h also increases with increasing D. However, it was explained in Section 6.4 that eqn (8) provides a conservative estimate for the minimum suitable h to use for a natural rock joint profile when the value of d is determined for the

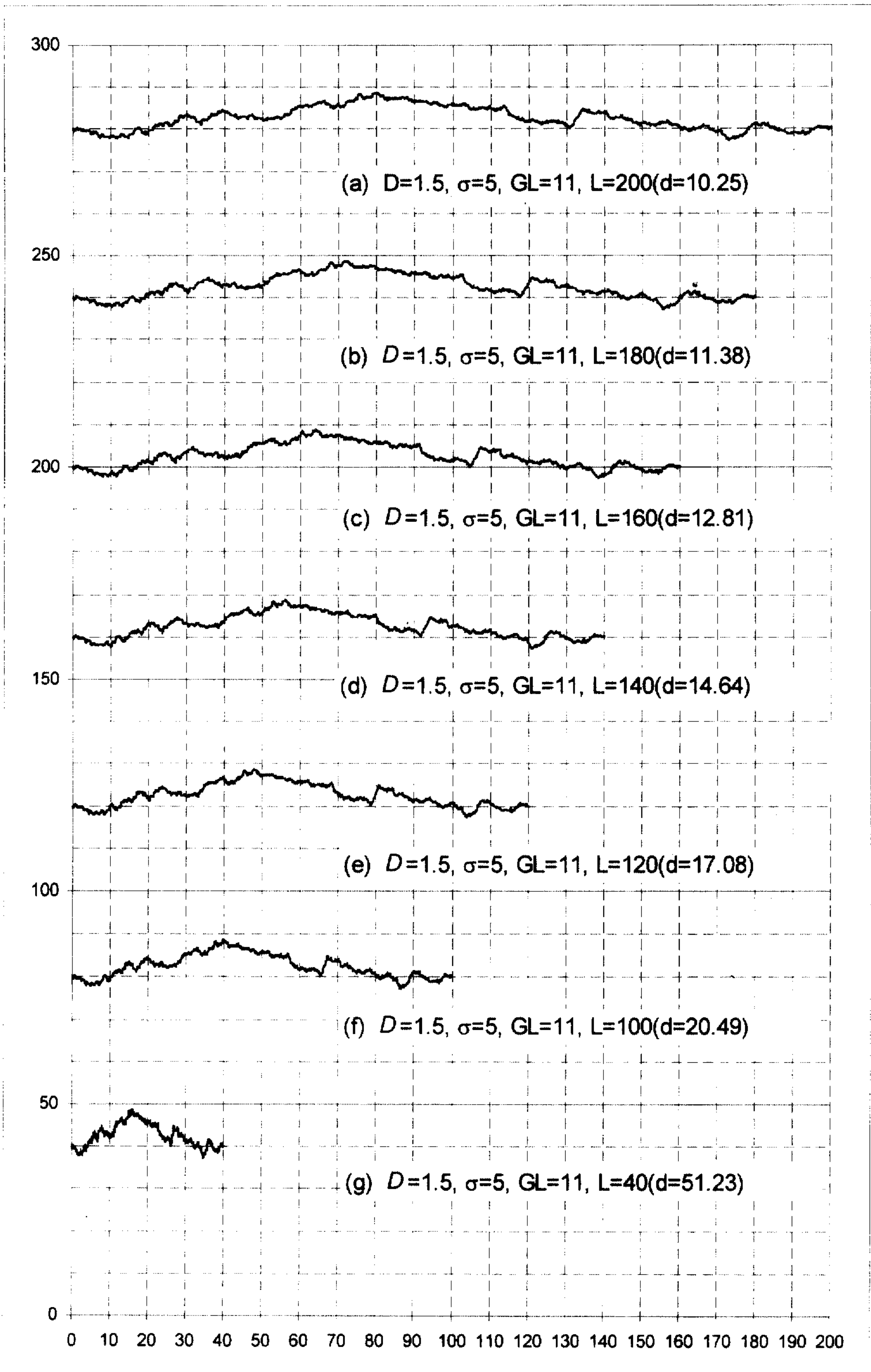


Fig. 15. Stationary fractional Brownian profiles with $D = 1.5$, $\sigma = 5$, $GL = 11$ and different profile lengths: (a) $L = 200$, (b) $L = 180$, (c) $L = 160$, (d) $L = 140$, (e) $L = 120$, (f) $L = 100$ and (g) $L = 40$.

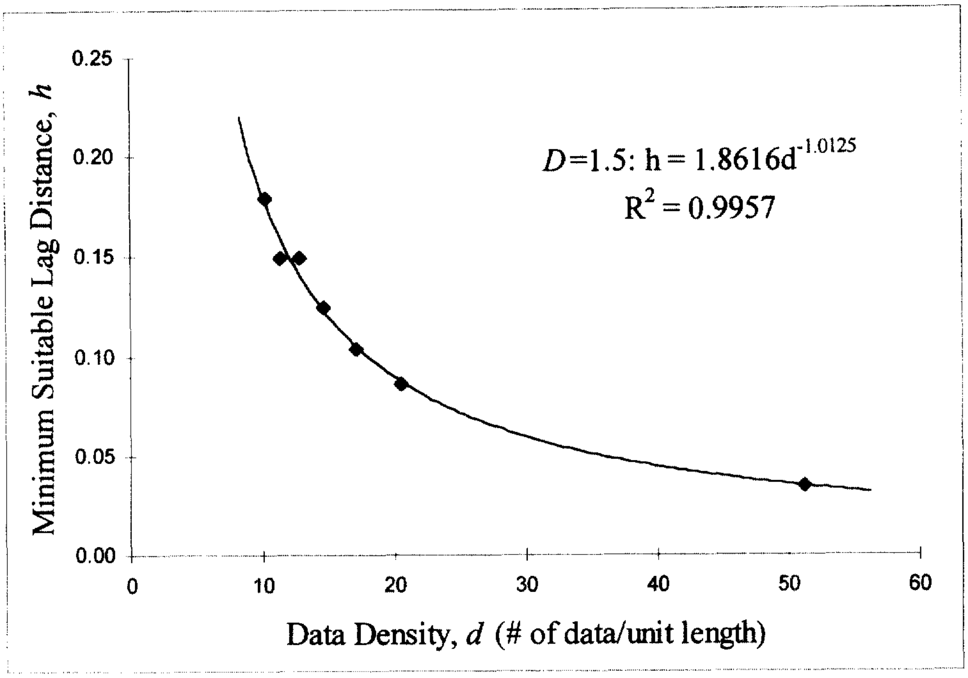


Fig. 16. Plot between minimum suitable lag distance and data density for stationary fractional Brownian profiles having $D = 1.5$, $GL = 11$ and different L.

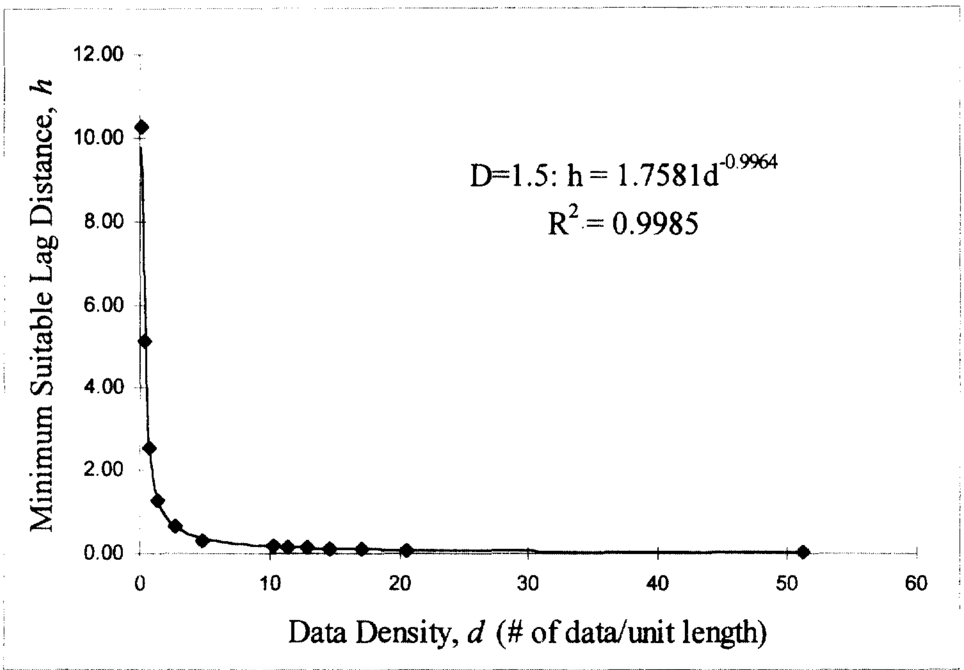


Fig. 17. Plot between minimum suitable lag distance and data density for stationary fractional Brownian profiles having $D = 1.5$.

profile. Then six more values of h can be calculated based on the estimated minimum h using an increment factor of 1.2. These seven values of h then can be used to estimate D and K_v accurately for the given natural rock joint profile.

Figure 20 shows that the estimated K_v increases with both D and σ for a fixed d value; effect of σ on K_v increases with increasing D. Figure 21 shows that K_v increases with d when

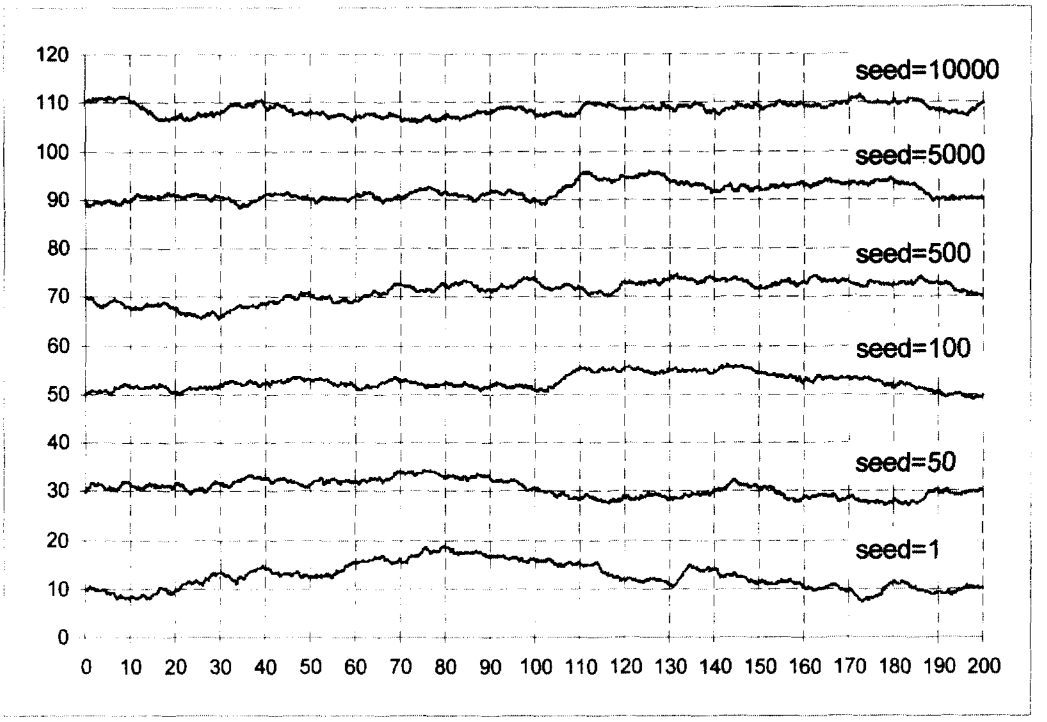


Fig. 18. Fractional Brownian profiles at the 11th generation level with $D = 1.5$, $\sigma = 5$, $d = 10.25$ and different seed values.

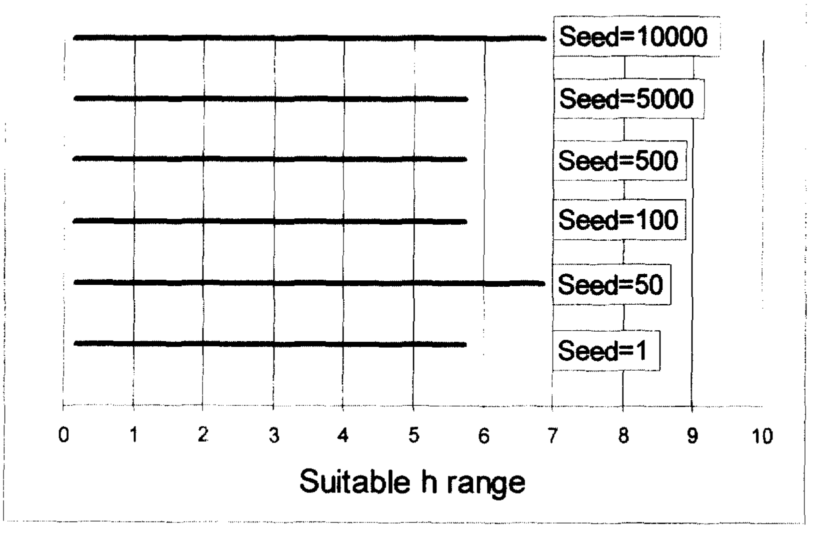


Fig. 19. Relation between suitable h range and seed value for fractional Brownian profiles having $D = 1.5$, $GL = 11$, $L = 200$ and $d = 10.25$.

both D and σ are kept constant. Multiple regression analysis performed on K_v, D, σ and d produced the following equation at an R^2 value of 0.9648:

$$K_v = 2.0 \times 10^{-5} d^{0.35} \sigma^{1.95} D^{14.5} \quad (9)$$

It is important to note that values for two parameters (D and σ) are used in generating a fractional Brownian profile. The aforementioned facts clearly indicate that at least two

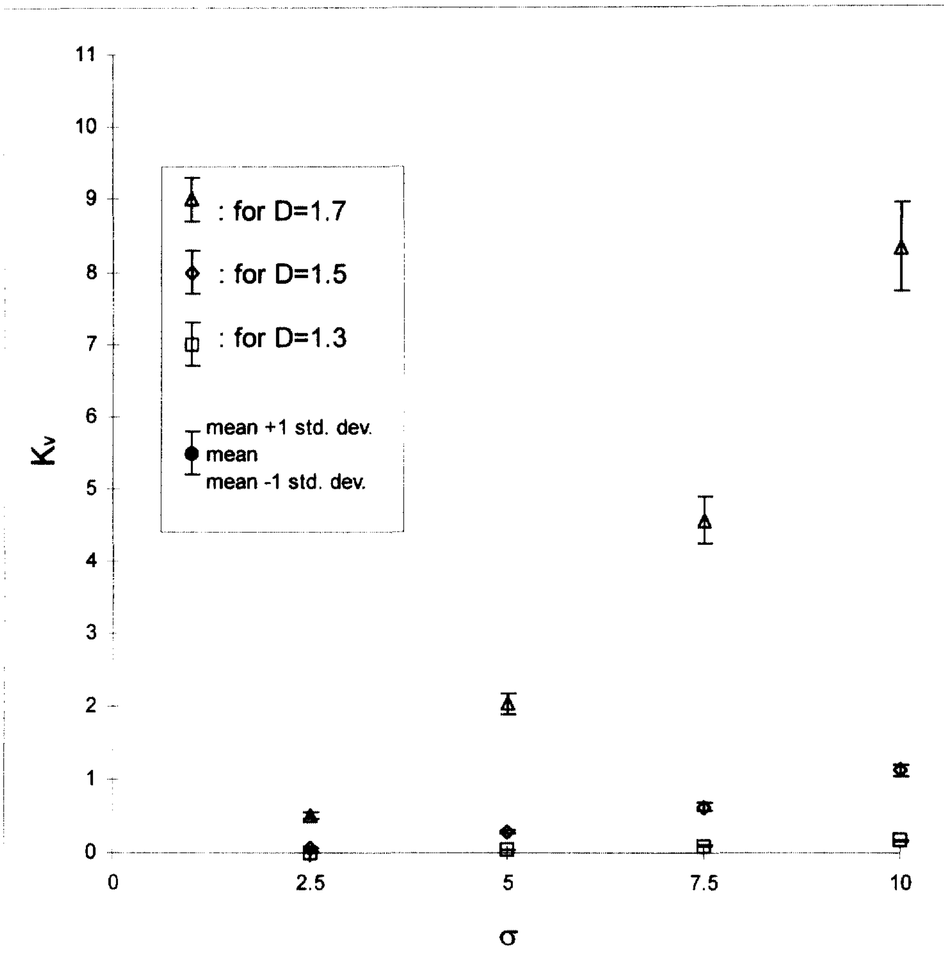


Fig. 20. Effect of D and σ on K_v for fractional Brownian profiles having $d = 10.25$.

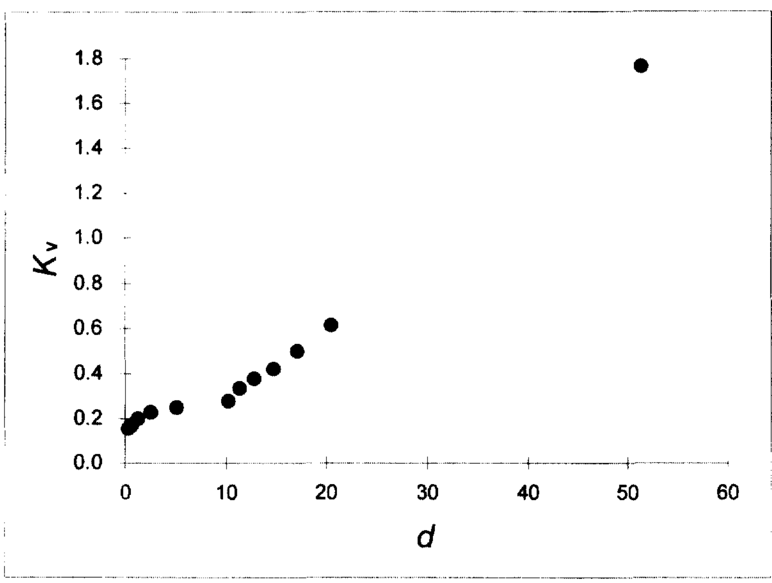


Fig. 21. Effect of d on K_v for fractional Brownian profiles having $D = 1.5$ and $\sigma = 5$.

fractal parameters are required to quantify stationary roughness. The fractal parameters D and K_v may be used in quantifying the stationary roughness with the variogram method. The parameter K_v can be used to capture the scale effect of roughness profiles. In addition, further parameters are required to quantify the non-stationarity of a roughness profile, if it exists. In the presence of a non-stationarity due to a global trend, in many cases, just the inclination or declination angle of the roughness profile in the direction considered would be sufficient to estimate the non-stationarity.

Acknowledgement—This research was funded by the U.S. National Science Foundation, Grant No. CMS-9522798.

REFERENCES

Barton, N. (1973) Review of a new shear strength criterion for rock joints. *Engrg. Geology* **7,** 287–332.

Berry, M. V. and Lewis, Z. V. (1980) On the Weierstrass-Mandelbrot fractal function. *R. Soc. Lond. Proc.*, Ser. A. 370, 459–484.

Brown, S. R. and Scholz, C. H. (1985) Broad band width study of the topography of natural rock surfaces. *J. Geophys. Res.* **90,** 12575–12582.

Den Outer, A., Kaashoek, J. F. and Hack, H. R. G. K. (1995) Difficulties with using continuous fractal theory for discontinuity surfaces. *Int. J. Rock Mech. Min. Sci.* **32,** 3–10.

Dight, P. M. and Chiu, H. K. (1981) Prediction of shear behaviour of joints using profiles. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstr.* **18,** 369–386.

Feder, J. (1988) *Fractals*. Plenum Press, New York.

Fournier, A., Fussel, D. and Carpenter, L. (1982) Computer rendering of stochastic models. *Commun. ACM* **25,** 371–384.

Fox, C. G. (1987) An inverse Fourier transform algorithm for generating random signals for a specified spectral form. *Comput. Geosci.* **13,** 369–374.

Huang, S. L., Oelfke, S. M. and Speck, R. C. (1992) Applicability of fractal characterization and modeling to rock joint profiles. *Int. J. Rock Mech. Min. Sci.* **29,** 89–98.

Krahn, J. and Morgenstern, N. R. (1979) The ultimate frictional resistance of rock discontinuities. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstr.* **16,** 127–133.

Kulatilake, P. H. S. W., Shou, G., Huang, T. H. and Morgan, R. M. (1995) New peak shear strength criteria for anisotropic rock joints. *Int. J. Rock Mech. Min. Sci.* **32,** 673–697.

Kulatilake, P. H. S. W. and Um, J. (1997) Requirements for accurate quantification of self-affine roughness using the roughness-length method. To appear in *Int. J. Rock Mech. and Min. Sci.*

Kulatilake, P. H. S. W., Um, J. and Pan, G. (1997) Requirements for accurate estimation of fractal parameters for self-affine roughness profiles using the line scaling method. To appear in *Rock Mech. and Rock Engrg.*

Maerz, N. H., Franklin, J. A. and Bennett, C. P. (1990) Joint roughness measurement using Shadow Profilometry. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstr.* **27,** 329–343.

Malinverno, A. (1990) A simple method to estimate the fractal dimension of a self affine series. *J. Geophys. Res. Lett.* **7,** 1953–1956.

Mandelbrot, B. B. (1967) How long is the coast of Britain? Statistical self similarity and fractal dimension. *Science* **155,** 636–638.

Mandelbrot, B. (1983) *The Fractal Geometry of Nature*. W. Freeman, San Francisco, p. 468.

Matsushita, M. and Ouchi, S. (1989) On the self affinity of various curves. *Physica* D, **38,** 246–251.

Miller, S. M., McWilliams, P. C. and Kerkering, J. C. (1990) Ambiguities in estimating fractal dimensions of rock fracture surfaces. *Proc. 31st U.S. Symp. on Rock Mech.* A.A. Balkema, Rotterdam, The Netherlands, pp. 471–478.

Odling, N. E. (1994) Natural fracture profiles, fractal dimension and joint roughness coefficients. *Rock Mech. and Rock Engrg.* **27,** 135–153.

Orey, S. (1970) Gaussian Sample Functions and Housdorff Dimension of Level Crossings. x. Wahrsheninkeits theorie verw. *Gebierte* **15,** 249–256.

Power, W. L. and Tullis, T. E. (1991) Euclidean and fractal models for the description of rock surface roughness. *J. Geophys. Res.* **96,** 415–424.

Reeves, M. J. (1990) Rock surface roughness and frictional strength. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstr.* **28,** 429–442.

Saupe, D. (1988) Algorithms for random fractals. In *The Science of Fractal Images*, ed. Peitgen and Saupe, pp. 71–113. Springer-Verlag, New York.

Shirono, T. and Kulatilake, P. H. S. W. (1997) Accuracy of the spectral method in estimating fractal/spectral parameters for self-affine roughness profiles. To appear in *Int. J. Rock Mech. and Min. Sci.*

Tse, R. and Cruden, D. M. (1979) Estimating joint roughness coefficients. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstr.* **16,** 303–307.

Voss, R. F. (1988) Fractals in nature: from characterization to simulation. In *The Science of Fractal Images*, ed. Peitgen and Saupe, pp. 22–69. Springer-Verlag, New York.

Wu, T. H. and Ali, E. M. (1978) Statistical representation of joint roughness. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstr.* **15,** 259–262.